

# Study of the $B_c$ -meson lifetime

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## Abstract

We in terms of optical theorem estimate the lifetime of  $B_c$ -meson with the parameters which are determined by fitting the data for the lifetimes and inclusive semilepton-decays of various  $B$  and  $D$  mesons. In the estimate, we find that the bound-state effects are important, and take them into account carefully in the framework which attributes the effects to the effective masses of the decay heavy quarks in the inclusive processes. We also find that to  $B_c$  lifetime the penguin contribution is enhanced due to possible interference between the penguin and the ‘tree part’  $c_1 O_1 + c_2 O_2$ .

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Very recently the meson  $B_c$  has been observed in CDF detector at Fermilab Tevatron. The observation is through the semi-leptonic decays  $B_c \rightarrow J/\psi + l + \nu_l$ , and not only the value of its mass  $m_{B_c} = 6.40 \pm 0.39 \pm 0.13$  GeV, but also the lifetime  $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$  ps are given [1]. Therefore to estimate its lifetime so as to understand the meson and its decay mechanisms becomes one re-freshed interesting problem.

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$B_c$ -meson is composed of two heavy flavors and both of them contribute to the lifetime comparatively. It is known that the Heavy Quark Effective Theory (HQET) [2] successfully applies to phenomenology of heavy mesons and baryons, although there are still some open problems in B and D physics, such as the unexpected difference between lifetimes of  $\Lambda_b$  and B-mesons [3] and missing charm puzzle [4] etc. Since there is no light flavor quarks in  $B_c$ -meson, HQET so the Isgur-Wise form factor scenario does not apply here.

Since the optical theorem may apply to inclusive processes properly so some non-perturbative effects can be absorbed by the theorem, thus we focus our attention on the inclusive processes, especially, the lifetime with the help of the theorem in this paper.

As realized by many authors, in D and B decays besides the spectator mechanism i.e. the direct decay of  $c$  or  $\bar{b}$ , the non-spectator mechanisms are quite important too. There are two distinct types of non-spectator mechanisms: the W-annihilation (WA) and the Pauli-interference (PI), whose details are depicted in [8]. As for  $B_c$  meson, owing to the non-spectator mechanisms the situation becomes more complicated and interesting. In  $B_{d(u)}, B_s$  decays, the penguin contributions and its interference with that of the ‘tree piece’  $c_1 O_1 + c_2 O_2$  can be ignored due to the CKM suppression. In contrary, for  $B_c$  meson,  $\bar{b} \rightarrow \bar{c} + “W^+”$  and  $“W^+” \rightarrow c + \bar{s}$  (here  $“W^+”$  denotes a virtual  $W^+$ -boson) are favored according to the CKM entries, so there are charm in initial state and charm, anti-charm in final state. Therefore, the interference between the penguin and ‘tree piece’ may contribute to the decay (see below for some details). Namely the penguin contribution becomes more important and  $B_c$ -meson decay may serve as an ideal place to study the penguin mechanism. Our numerical results show that such interference can cause a contribution as large as about 3~4% in the total width, while in  $B_{d(u)}$  and  $B_s$  cases, the contribution is less than 0.5%.

Moreover, in  $B_c$ , as in the heavy quarkonia, the bound-state effects should be considered carefully, even when evaluating its inclusive processes. We take a phenomenological approach to deal with the masses of the decay quarks  $b(\bar{b})$  and  $c(\bar{c})$  in various heavy mesons i.e. in various

bound-state by fitting data[5]. Recently, a modification of HQET, the so-called Heavy Quark Effective Field Theory (HQEFT), is proposed[6], where the bound state effects are taken into account for  $B-$  hadrons: when evaluating the decays, the b-quark mass takes a different value in  $\Lambda_b$  from that in B-mesons, so the aforementioned problem, the lifetime difference between  $\Lambda_b$  and  $B-$ mesons, can be answered reasonably. Indeed, the bound state effects, which affect the inclusive decays, may be attributed to the effective mass of the decay quark mainly.

For  $B_{d(u)}, B_s$  decays, the contributions to the decay widths from  $\bar{b}$  and  $c$  quarks are described quite well in literature[7, 9, 10], and the general formulation for the non-spectator contributions WA and PI have been given in [8]. The formulation of  $B_c$  is given in [5] and the readers who are interested in the details are advised to refer to it. Here we only present the part of formulation relating to  $B_c$  for later convenience, focus on description of the physical picture, and discuss the physical essence and consequences.

The spectator contributions to  $B_c$  lifetime are the incoherent sum of  $\bar{b}$  and  $c$  decay, when the bound-state effects, as pointed out above, are attributed to the effective masses of  $\bar{b}$  and  $c$  quarks respectively only, that is one of essential differences from the other  $B-$ mesons.

$$\Gamma^{\text{spectator}} = \Gamma_b^{\text{spectator}} + \Gamma_c^{\text{spectator}}. \quad (1)$$

Furthermore, the non-spectator parts may have a unignorable interference.

The effective Lagrangian, on that the present study is based, is

$$\begin{aligned} L_{eff}^{\Delta B=1} = & -\frac{4G_F}{\sqrt{2}} \left\{ V_{cb}[V_{ud}^*(c_1(\mu)O_1^u + c_2(\mu)O_2^u) + V_{cs}^*(c_1(\mu)O_1^c + c_2(\mu)O_2^c) + \right. \\ & \left. \sum_{l=e,\tau,\mu} \bar{l}\gamma_\mu L\nu\bar{c}\gamma^\nu Lb + V_{cs}^* \sum_{i=3}^6 c_i O_i \right\} + h.c. \end{aligned} \quad (2)$$

Here the notations for operators and their coefficients are given in [11].

Skipping the tedious details, we have the contribution of PI i.e.  $\Gamma^{PI}(\text{tree})$  and  $\Gamma^{PI}(\text{penguin})$  to the total width as

$$\Gamma^{PI}(\text{tree}) = \frac{G_F^2}{4\pi} f_{B_c}^2 M_{B_c} |V_{cb}|^2 |V_{cs}|^2 (1 - z_-)^2 p_-^2 \cdot \left\{ [2c_1 c_2 + \frac{1}{N}(c_1^2 + c_2^2)] B_1 \right.$$

$$+2(c_1^2 + c_2^2)\epsilon_1\} \quad (3)$$

$$\begin{aligned} \mathbf{\Gamma}^{PI}(\text{penguin}) = & \frac{G_F^2}{4\pi} f_{B_c}^2 M_{B_c} |V_{cb}|^2 |V_{cs}|^2 (1 - z_-)^2 p_-^2 \cdot \left\{ [2c_2 c_4 + 2c_1 c_3 + 2c_3 c_4 + \right. \\ & \left. \frac{1}{N}(c_3^2 + c_4^2 + 2c_2 c_3 + 2c_1 c_4)] B_1 + 2(c_3^2 + c_4^2 + 2c_2 c_3 + 2c_1 c_4) \epsilon_1 \right\} \end{aligned} \quad (4)$$

$$\begin{aligned} & - \frac{G_F^2}{4\pi} f_{B_c}^2 M_{B_c} |V_{cb}|^2 |V_{cs}|^2 (1 - z_-)^2 \cdot \left\{ [2c_5 c_6 + \frac{1}{N}(c_5^2 + c_6^2)] [\frac{2+z_-}{3} p_-^2 \tilde{B}_2 \right. \\ & - \frac{1+2z_-}{6} (m_b^2 \tilde{B}_1 + m_c^2 B_1 - 4m_b m_c B_2 + 2m_b m_c B_1)] + 2(c_5^2 + c_6^2) [\frac{2+z_-}{3} p_-^2 \tilde{\epsilon}_2 \\ & - \frac{1+2z_-}{6} (m_b^2 \tilde{\epsilon}_1 + m_c^2 \epsilon_1 - 2m_b m_c (\epsilon_3 + \epsilon_4) + m_b m_c (\epsilon_5 + \epsilon_6))] \Big\} \\ & - \frac{G_F^2}{8\pi} f_{B_c}^2 M_{B_c} |V_{cb}|^2 |V_{cs}|^2 (1 - z_-)^2 \bar{m}_c \cdot \left\{ [c_1 c_5 + c_2 c_6 + c_3 c_6 + c_4 c_5 \right. \\ & + \frac{1}{N}(c_1 c_6 + c_2 c_5 + c_4 c_6 + c_3 c_5)] [2m_c B_1 + m_b (-4B_2 + 2B_1)] \\ & \left. + 2(c_1 c_6 + c_2 c_5 + c_4 c_6 + c_3 c_5) [2m_c \epsilon_1 - 2m_b (\epsilon_3 + \epsilon_4) + m_b (\epsilon_5 + \epsilon_6)] \right\}. \end{aligned} \quad (5)$$

For WA, we have three pieces as the follows:

$$\begin{aligned} \mathbf{\Gamma}^{WA}(\text{tree}) = & - \frac{G_F^2}{12\pi} |V_{cb}|^2 |V_{cs}|^2 f_{B_c}^2 M_{B_c} (1 - z_+)^2 \cdot \left\{ [Nc_1^2 + 2c_1 c_2 + \frac{c_2^2}{N}] \right. \\ & \times [(1 + \frac{z_+}{2}) M_{B_c}^2 B_1 - (1 + 2z_+) (m_b^2 B_2 + m_c^2 \tilde{B}_2 + 2m_b m_c B_2)] \\ & \left. + 2c_2^2 [(1 + \frac{z_+}{2}) M_{B_c}^2 \epsilon_1 - (1 + 2z_+) (m_b^2 \epsilon_2 + m_c^2 \tilde{\epsilon}_2 + m_b m_c (\epsilon_3 + \epsilon_4))] \right\}, \end{aligned} \quad (6)$$

$$\begin{aligned} \mathbf{\Gamma}^{WA}(B_c \rightarrow \tau \nu) = & - \frac{G_F^2}{12\pi} |V_{cb}|^2 |V_{cs}|^2 f_{B_c}^2 M_{B_c} (1 - z_+)^2 \cdot \left\{ (1 + \frac{z_\tau}{2}) M_{B_c}^2 B_1 \right. \\ & \left. - (1 + 2z_\tau) (m_b^2 B_2 + m_c^2 \tilde{B}_2 + 2m_b m_c B_2) \right\}, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathbf{\Gamma}^{WA}(\text{penguin}) = & - \frac{G_F^2}{12\pi} |V_{cb}|^2 |V_{cs}|^2 f_{B_c}^2 M_{B_c} (1 - z_+)^2 \cdot \left\{ [(\frac{2c_2 + c_3}{N} + 2c_1 + c_4) (c_3 + Nc_4)] \right. \\ & \times [(1 + \frac{z_+}{2}) M_{B_c}^2 B_1 - (1 + 2z_+) (m_b^2 B_2 + m_c^2 \tilde{B}_2 + 2m_b m_c B_2)] \\ & + 2(2c_2 + c_3) c_3 \cdot [(1 + \frac{z_+}{2}) p_+^2 \epsilon_1 - (1 + 2z_+) (m_b^2 \epsilon_2 + m_c^2 \tilde{\epsilon}_2 + m_b m_c (\epsilon_3 + \epsilon_4))] \Big\} \\ & + \frac{G_F^2}{2\pi} |V_{cb}|^2 |V_{cs}|^2 f_{B_c}^2 M_{B_c}^3 (1 - z_+)^2 \cdot \left\{ [\frac{c_5^2}{N} + 2c_5 c_6 + Nc_6^2] \tilde{B}_2 + 2c_5^2 \tilde{\epsilon}_2 \right\} \\ & - \frac{G_F^2}{4\pi} |V_{cb}|^2 |V_{cs}|^2 f_{B_c}^2 M_{B_c} \bar{m}_c (1 - z_+)^2 \cdot \left\{ [(\frac{c_2 + c_3}{N} + c_1 + c_4) (c_5 + Nc_6)] \right. \\ & \left. \times [2m_b B_2 + 2m_c \tilde{B}_2] + 2(c_2 + c_3) c_5 \cdot [m_b (\epsilon_3 + \epsilon_4) + 2m_b \tilde{\epsilon}_2] \right\}, \end{aligned} \quad (8)$$

where

$$z_- = \frac{\bar{m}_c^2}{p_-^2}, \quad p_- = p_b - p_{\bar{c}}, \quad (9)$$

and

$$\begin{aligned} p_+ &= p_b + p_c, \\ z_+ &= \frac{\bar{m}_c^2}{p_+^2} = \frac{\bar{m}_c^2}{M_{B_c}^2}, \\ z_\tau &= \frac{m_\tau^2}{p_+^2} = \frac{m_\tau^2}{M_{B_c}^2}. \end{aligned} \quad (10)$$

Note that due to the chiral suppression only  $B_c \rightarrow \tau^+ \nu$  and  $B_c \rightarrow c \bar{s}$  are considered for WA.

In the expressions the pieces with the superscripts ‘WA’ or ‘PI’ (corresponding to the W-annihilation or Pauli-interference parts) and *tree* in the brackets denote all the contributions from the ‘tree’ part, i.e. the  $c_1 O_1$  and  $c_2 O_2$  terms, whereas those with *penguin* in the brackets mean the contributions not only from the *penguin* part  $c_i O_i$  ( $i = 3, 4, 5, 6$ ) alone, but also from its interference with the ‘tree’ part. Obviously, for  $B_c$ , there are interferences proportional to  $c_i c_1, c_i c_2$  ( $i = 3 \dots 6$ ), whereas for  $B_{d(u)}, B_s$ , proportional to  $c_i c_j$  ( $i, j = 3, \dots 6$ ), the interferences among the penguin terms themselves only. Since  $c_1, c_2$  are of order  $O(1)$  and  $c_i$  ( $i = 3, \dots 6$ ) is of order  $O(10^{-2})$ , so roughly,  $c_i c_j$  is four orders smaller than  $c_1^2, c_2^2, c_1 c_2$ , but  $c_i c_1, c_i c_2$  are only two (even one) orders smaller, thus the penguin terms may play more significant roles in  $B_c$  decays. Moreover, note that since  $m_c$  cannot be neglected in the derivation of decay-width operators for  $B_c$  decays, while the light flavors are taken as zero-mass fermions for  $B_{d(u)}, B_s$  decays, so several new operators appear.

To calculate the the contributions of the non-spectator parts, one needs to evaluate the hadronic matrix elements with dimension 6 operators even for the lifetime, which are governed by the non-perturbative QCD, and so far can be dealt with phenomenologically only. As pointed out by many authors [8], the non-factorization effects modify the results of the ‘vacuum saturation’ and the modifications can be described by introducing in a few parameters  $B_k$  (deviating from

1) and  $\epsilon_k$  (deviating from 0)<sup>2</sup>. With some symmetry arguments as in literature, we fix the values for the parameters so the hadronic matrix elements within tolerable errors.

As aforementioned, Instead of determining the heavy quark masses in bound-states from any underlying theory as what [6] did, we phenomenologically take the bound state effects into account of the quark masses by fitting data of the lifetimes and branching ratios of inclusive-semileptonic decays for the mesons  $B_{d(u)}$  and  $B_s$ .

We simply write

$$M_Q^{eff} = M_Q^{pole} - \Delta, \quad (11)$$

where  $\Delta$  manifests the bound-state effects and will be fixed phenomenologically.

Fitting the data for  $B_{d(u)}$  and  $B_s$  as well as D-mesons, we obtain:

$$\Delta_c^{(D)} \equiv m_c^{pole} - m_c^{eff} = 0.23 \text{ GeV};$$

$$\Delta_b^{(B)} \equiv m_b^{pole} - m_b^{eff} = 0.11 \sim 0.13 \text{ GeV}.$$

The superscripts (B) and (D) denote that the  $\Delta$  for b and c-quarks are determined by fitting the data of D and B mesons respectively.

For the  $B_c$  meson, the spectator contribution is an incoherent sum of that from individual  $\bar{b}$  and  $c$  quark decays while leaving the other as a spectator. But we should note that when evaluating this contribution,  $m_b$  and  $m_c$  take their effective values at the energy scales  $M_B \sim m_b(m_b)$  for the pole mass  $m_b^{pole}(B_c)$  and at  $M_D \sim m_c(m_c)$  for the pole mass  $m_c^{pole}(B_c)$ . Namely, we determine the effective masses for  $\bar{b}$  and  $c$  of the  $B_c$ -meson through  $B$  and  $D$  decays. Whereas the  $\bar{c}$  in the final state is the decay product of  $\bar{b}$ -quark and now we are considering inclusive processes, so we should take its running mass for the  $\bar{c}$ -quark at the energy scale of  $M_B$ . For the non-spectator contributions, i.e. the WA and PI pieces, the corresponding scale for the anti-charm quark mass of the final state should be  $M_{B_c}$ . In the numerical calculations for the

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<sup>2</sup>In fact in Eqs.(3-8) the modifications from the vacuum saturation have been made in this way already. The detailed expressions can be found in [5]

present paper, we take the relevant parameters as follows:  $M_{B_c} = 6.25$  GeV,  $M_{B_c}^* = 6.33$  GeV,  $B_i = \tilde{B}_i (i = 1, 2) = 1$ ,  $\epsilon_1 = \tilde{\epsilon}_1 = -0.14$ ,  $\epsilon_2 = \tilde{\epsilon}_2 = -0.08$ ,  $\epsilon_{3,4} \sim \epsilon_2$ ,  $\epsilon_{5,6} \sim \epsilon_1$ , and for the decay constant we adopt two possible values  $f_{B_c} = 500$  MeV[12] and  $f_{B_c} = 440$  MeV[13] respectively. For the calculation of PI contribution,  $p_-^2 = (p_{\bar{b}} - p_c)^2 \sim (2m_b^2 + 2m_c^2 - M_{B_c}^2)$  is adopted. With the parameters, we finally obtain the numerical results and tabulate them in Table 1.

	$\tau_{B_c}$ (ps)	$\Gamma^{pen.}$	$\Gamma^{b \rightarrow c}$	$\Gamma^{c \rightarrow s}$	$\Gamma^{WA}$	$\Gamma^{PI}$	$\Gamma(\tau\nu)(\text{ps}^{-1})$	$B_{SL}$
$f_{B_c} = 440\text{MeV}$	0.362	3.4%	22.8%	70.9%	13.4%	-7.1%	0.078	8.7%
$f_{B_c} = 500\text{MeV}$	0.357	4.3%	22.4%	69.7%	16.9%	-9.0%	0.100	8.4%

Table 1: This is the result for  $B_c$  meson,  $\tau_{B_c}$  denotes the lifetime of  $B_c$ ,  $\Gamma^{pen.}$  denotes the enhancement caused by the penguin contribution. The  $\Gamma(\tau\nu)$  denotes the width of the total leptonic decay, and the  $B_{SL}$  denotes the branching ratio of the semileptonic decay of  $B_c$ .

Here the puzzle about the effective masses of  $\bar{b}$  and  $c$  emerges again, because both of them reside in the same bound state  $B_c$ . Taking all the above parameters, we would obtain the values presented in table 1. Now let us consider the bound-state effects on the effective quark-masses in  $B_c$  meson more precisely. Because  $B_c$  includes two heavy quarks, the bound-state effects might be different from those in  $B, D$  mesons which contains one light flavor. If we are tempted to believe that the values  $m_c^{eff}$  and  $m_b^{eff}$  might be smaller than  $m_c^{eff} = 1.65$  GeV and  $m_b^{eff} = 4.9$  GeV obtained by  $B$  and  $D$  decays and phenomenologically if  $m_c^{eff}(B_c) = 1.55$  GeV,  $m_b^{eff}(B_c) = 4.85$  GeV instead, we would have  $\tau(B_c) \approx 0.47$  ps, which is closer to the recently measured  $B_c$  lifetime[1]. In this case, it will mean  $\Delta_c^{B_c} = 0.33$  GeV and  $\Delta_b^{B_c} = 0.17$  GeV.

The results indicate the importance of the bound-state effects on the phenomenology. It is easy to understand the fact, i.e. the rates of direct  $\bar{b}$  and  $c$  decays, which dominate the lifetime of  $B_c$  meson, are proportional to  $(m_Q^{eff})^5$ , the results are somewhat sensitive to the  $m_Q^{eff}$ -values. It is a ‘good’ place to ‘determine’ the quark effective masses  $m_Q$ . From the physics picture, the quark effective masses introduced here should have more precise meaning in the framework of

the potential model and be good to involve certain non-perturbative QCD effects[14].

As a summary, in this work, we study the effects when decaying quarks reside in bound states phenomenologically with care. We collect all the informations gained from heavy mesons  $B_{d(u)}$ ,  $B_s$  and  $D$ , we further calculate the lifetime of  $B_c$  meson. In the process, we notice that the bound state effects are important even to the spectator modes. Moreover, because of the CKM entries, and the interference between the penguin and the tree piece, the penguin contribution is more important in  $B_c$  case than in  $B_{d(u)}$  and  $B_s$ . Therefore, further experimental progress in measuring  $B_c$  lifetime and branching ratios may provide more informations about the reaction mechanisms and especially it would make definite hints to the interesting penguin mechanism etc.

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